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Two-hadron interference fragmentation functions Part II: a model calculation

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We present a model calculation of leading order interference fragmentation functions arising from the distribution of two hadrons produced in the same jet of a fragmenting quark in a hard process. Using a simple spectator model for the mechanism of the hadronization process, for the first time we give a complete calculation and numerical estimates for the example of a proton-pion pair production with its invariant mass on the Roper resonance.

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I. INTRODUCTION

In a companion paper [1] we have discussed the general framework of two-hadron interference fragmentation functions, their definitions and properties. We have given the cross section of two-hadron inclusive lepton-nucleon scattering as an example how they can be disentangled experimentally by using azimuthal angular dependences of a hard process. In the present article we supplement our investigation with a spectator model calculation of the interference fragmentation functions for the special case of the hadron-pair in the quark jet being a pion and a proton.

Fragmentation functions (FF), like distribution functions (DF), ultimately contain the complete information on the confinement of partons inside hadrons. Factorization theorems for hard processes, whenever available, ensure their universality, i.e. FF (and DF) are process independent; once determined in a hard process they can be used to predict observables in other hard reactions. FF and DF describe the properties of partons in hadronic bound states at a low scale. Their intrinsic non-perturbative nature presently prevents a complete calculation from first principles within the framework of Quantum Chromodynamics. On the experimental side, several of the DF are nowadays quite well known from deep inelastic inclusive lepton-nucleon scattering, the Drell-Yan process or from jet production in nucleon-nucleon collisions. The experimental knowledge of the various FF, however, is very poor because it requires the ability of measuring more exclusive channels in hard processes (semi-inclusive cross sections, transverse momenta and/or polarizations of detected hadrons, etc..). Therefore, model calculations of FF are of particular importance to learn about their possible features and to estimate cross sections for future experiments.

When new channels are open in the final state, several FF arise which can also be time-reversal odd [2,3] (for brevity, "T-odd"), in the sense that the constraints due to time-reversal invariance cannot be applied. The main reason for that is the existence of residual Final State Interactions (FSI) between the produced hadron(s) and the remnants of the fragmenting quark. Thus, FF not only contain information complementary to the one given by DF, but in particular the T-odd ones also represent a powerful tool to explore processes taking place inside the jet. Moreover, specific T-odd interference FF can represent the necessary chiral odd partner to address the transversity distribution h_1 [4]. Therefore, the aim of the present paper is to calculate T-odd interference FF within a model.

The presence of FSI means that in the fragmentation process there are at least two leading interfering channels. In the previous paper [1] we have shown that the interference of two channels is not enough to generate "T-odd" FF. For example, in the case of one-hadron semi-inclusive processes we can model FSI by introducing an external potential. Despite this simplistic approach, we encounter a serious mathematical difficulty, because the potential in principle breaks the translational and rotational invariance of the problem. Simplifying assumptions about the symmetry of the potential can be introduced, but at the price of loosing any contribution to the "T-odd" structure of the amplitude. Alternatively, we have explicitly shown that a genuine difference in the Lorentz structure of the vertices describing the two interfering processes is needed to produce

a "T-odd" contribution [1]. All this means that addressing "T-odd" FF in one-hadron semi-inclusive processes requires the ability of describing the FSI between the outgoing hadron and the remnant of the jet, relating the modifications of the hadron wave function to a realistic microscopic description of the fragments.

Because of these arguments, a more convenient way to investigate T-odd FF is to look at the hadronization of a current quark where two leading hadrons are detected within the same jet, considering the remnant of the jet as a spectator and summing over all its possible configurations. By interference of different channels producing the two hadrons, FF emerge which are "T-odd", and can be both chiral even or chiral odd [1]. For the case of the two hadrons being a pair of pions the resulting FF have been proposed to investigate the transverse spin dependence of fragmentation. Collins and Ladinsky [5] considered the interference of a scalar resonance with the channel of independent successive two pion production. Jaffe, Jin and Tang [6] proposed the interference of s- and p-wave production channels, where the relevant phase shifts are essentially known. We will estimate the FF in the case of the pair being a proton and a pion produced either through non-resonant channels or through the Roper (1440 MeV) resonance.

This paper is organized as follows. In Sec. II a brief summary of the definitions, properties and relevant formulae about two-hadron FF are given together with a description of the kinematics. We restrict ourselves to the information necessary to keep this paper self-contained; full details can be found in Ref. [1]. In Sec. III we discuss an extended version of the spectator model used in Ref. [7–9] and here adopted for the calculation. In Sec. IV numerical estimates are presented and discussed for "T-odd" FF that emerge to leading order. Finally, a brief summary is given in Sec. V.

II. FRAGMENTATION FUNCTIONS FOR PROTON-PION PAIR PRODUCTION

As discussed in some detail in the companion paper [1] the two-hadron FF can be defined as Dirac projections of (partly integrated) quark-quark correlation functions. In the field theoretical description of hard processes those functions describe the soft parts which connect quark lines to hadrons, i.e. they are hadronic matrix elements of non-local operators built from quark (or gluon) fields. For a quark fragmenting into a jet which contains a pion-proton pair the appropriate correlation function (in a light-cone gauge) is

$$\Delta_{ij}(k; P_p, P_\pi) = \sum\!\!\!\!\!\!\!\int_X \int \frac{d^4\zeta}{(2\pi)^4}\, e^{ik\cdot\zeta}\, \langle 0|\psi_i(\zeta)|\pi, p, X\rangle\langle X, p, \pi|\overline{\psi}_j(0)|0\rangle \tag{1}$$

where the sum runs over all the possible intermediate states containing the pair. The momenta are indicated in Fig. 1.

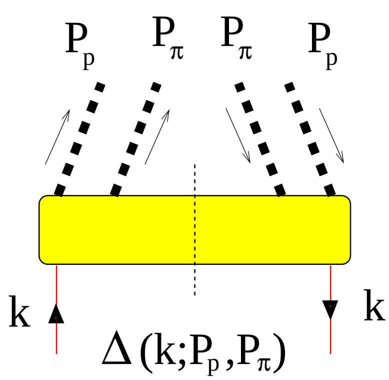


FIG. 1. Quark-quark correlation function for the fragmentation of a quark into a proton-pion pair.

Before proceeding with the definition of FF obtained from Δ, we summarize the discussion of kinematics as given in [1]. For clarity we give an explicit parametrization in a reference frame where the sum of proton and pion momenta $P_h = P_p + P_\pi$ has no transverse components. Two dimensionless light-like vectors n_+ and n_- satisfying $n_+ \cdot n_- = 1$ can be used to decompose a generic 4-vector a in its light-cone components $a^\pm = \left(a^0 \pm a^3\right)/\sqrt{2} = a.n_\mp$ and a two-dimensional transverse vector $\vec{a}_T$. Throughout the paper we use the notation $a^\mu = [a^-, a^+, \vec{a}_T]$. A possible parametrization of the momenta is

$$
\begin{aligned}
k &= \left[\frac{P_h^-}{z_h} \; , \; z_h \frac{k^2 + \vec{k}_T^2}{2 P_h^-} \; , \; \vec{k}_T \right] \; , \\
P_p &= \left[\xi \, P_h^- \; , \; \frac{M_p^2 + \vec{R}_T^2}{2 \, \xi \, P_h^-} \; , \; \vec{R}_T \right] \; , \\
P_\pi &= \left[(1-\xi) \, P_h^- \; , \; \frac{M_\pi^2 + \vec{R}_T^2}{2 \, (1-\xi) \, P_h^-} \; , \; -\vec{R}_T \right] \; ,
\end{aligned} \tag{2}
$$

where $R \equiv (P_p - P_\pi)/2$ is (half of) the relative momentum between the proton and the pion, and we defined the light-cone momentum fractions

$$
z_h = \frac{{P_h}^-}{k^-} \; , \qquad \xi = \frac{P_p^-}{P_h^-} \; , \qquad (1-\xi) = \frac{P_\pi^-}{P_h^-} \; . \tag{3}
$$

With the constraint of reproducing on-shell hadrons at fixed mass ($P_p^2 = M_p^2, P_\pi^2 = M_\pi^2$), the dependence of Δ on the momenta k, P_p, P_π in Eq. (1) can be reexpressed in terms of seven independent variables: the light-cone component of the hadron pair momentum, P_h^-; the light-cone fraction of the quark momentum carried by the hadron pair, $z_h = P_h^-/k^-$; the fraction of hadron pair momentum carried by each individual hadron, ξ; the four independent invariants that can be formed in this case, i.e.

$$
\begin{aligned}
& \tau_h = k^2 \; , \\
& \sigma_h = 2k \cdot P_h = \left\{ \frac{M_p^2 + \vec{R}_T^2}{z_h \, \xi} + \frac{M_\pi^2 + \vec{R}_T^2}{z_h \, (1-\xi)} \right\} + z_h \, (\tau_h + \vec{k}_T^2) \; , && \text{(4a)} \\
& \sigma_d = 2k \cdot (P_p - P_\pi) = \left\{ \frac{M_p^2 + \vec{R}_T^2}{z_h \, \xi} - \frac{M_\pi^2 + \vec{R}_T^2}{z_h \, (1-\xi)} \right\} + z_h (2\xi - 1)(\tau_h + \vec{k}_T^2) - 4 \, \vec{k}_T \cdot \vec{R}_T \; , && \text{(4b)} \\
& M_h^2 = P_h^2 = 2 \, P_h^+ \, P_h^- = \left\{ \frac{M_p^2 + \vec{R}_T^2}{\xi} + \frac{M_\pi^2 + \vec{R}_T^2}{1-\xi} \right\} \; , && \text{(4c)}
\end{aligned}
$$

where Eq. (4c) can also be expressed as

$$
\vec{R}_T^2 = \xi \, (1-\xi) \, M_h^2 - (1-\xi) \, M_p^2 - \xi \, M_\pi^2 \; . \tag{5}
$$

In the following definitions of the FF the correlation function Δ will occur integrated over the (hard-scale) suppressed light-cone component k^+ at $k^- = P_h^-/z_h$ and traced with a certain Dirac structure Γ in the form

$$
\Delta^{[\Gamma]} = \frac{1}{4 z_h} \int dk^+ \int dk^- \, \delta \left(k^- - \frac{P_h^-}{z_h} \right) \, \mathrm{Tr}[\Delta \Gamma] \; . \tag{6}
$$

This will reduce the number of independent variables to the five $z_h, \xi, \vec{k}_T^{\,2}, M_h^2, \sigma_d$. To make this evident we rewrite the integration in a covariant way using

$$
2 \, P_h^- = \frac{d\sigma_h}{dk^+}, \qquad 2 \, k^+ = \frac{d\tau_h}{dk^-}, \tag{7}
$$

and the relation

$$
\frac{1}{2k^+} \delta \left(k^- - \frac{P_h^-}{z_h} \right) = \delta \left(2k^+ k^- - \frac{2k^+ P_h^-}{z_h} \right) = \delta \left(\tau_h + \vec{k}_T^{\,2} - \frac{\sigma_h}{z_h} + \frac{M_h^2}{z_h^2} \right) \; . \tag{8}
$$

We get the final expression

$$
\Delta^{[\Gamma]}(z_h, \xi, \vec{k}_T^{\,2}, M_h^2, \sigma_d) = \int d\sigma_h \, d\tau_h \, \delta \left(\tau_h + \vec{k}_T^{\,2} - \frac{\sigma_h}{z_h} + \frac{M_h^2}{z_h^2} \right) \, \frac{\mathrm{Tr}[\Delta(z_h, \xi, P_h^-, \tau_h, \sigma_h, M_h^2, \sigma_d) \, \Gamma]}{8 z_h P_h^-} \; , \tag{9}
$$

where the dependence on the transverse quark momentum $\vec{k}_T^{\,2}$ through σ_h is made explicit by means of Eqs. (4a) and (5). Using the parametrization Eq. (4) it is possible to further reexpress the set of kinematical variables

as z_h, ξ, $\vec{k}_T^2$ and $\vec{R}_T^2$, $\vec{k}_T \cdot \vec{R}_T$, where $\vec{R}_T$ is (half of) the transverse momentum between the two hadrons in the considered frame.

Two-hadron FF to leading order are defined by the following Dirac projections

$$\Delta^{[\gamma^-]}(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) = D_1(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) \tag{10}$$

$$\Delta^{[\gamma^- \gamma_5]}(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} G_1^\perp(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) \tag{11}$$

$$\begin{aligned}\Delta^{[i\sigma^{i-} \gamma_5]}(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) = &\frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^{\sphericalangle}(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) \\ &+ \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^\perp(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T \cdot \vec{R}_T) ,\end{aligned} \tag{12}$$

where we used transverse 4-vectors defined as $a_T^\mu = g_T^{\mu\nu} a_\nu = [0, 0, \vec{a}_T]$ (with $g_T^{\mu\nu} = g^{\mu\nu} - n_+^\mu n_-^\nu - n_+^\nu n_-^\mu$). In this manner, the FF depend on how much of the fragmenting quark momentum is carried by the hadron pair (z_h), on the way this momentum is shared inside the pair (ξ), and on the "geometry" of the pair, namely on the relative momentum of the two hadrons ($\vec{R}_T^2$) and on the relative orientation between the pair plane and the quark jet axis ($\vec{k}_T^2$, $\vec{k}_T \cdot \vec{R}_T$, see also Fig. 2).

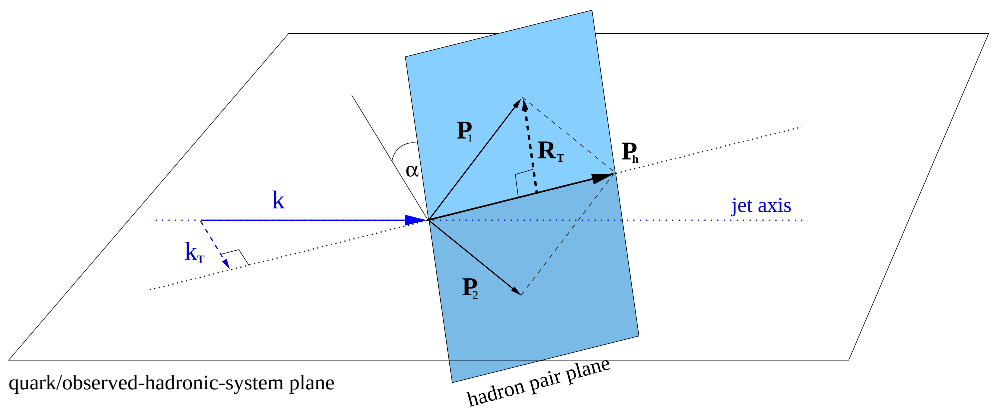


FIG. 2. Kinematics for a fragmenting quark jet containing a pair of leading hadrons.

In Eqs. (10)-(12), $D_1, G_1^\perp, H_1^\perp, H_1^{\sphericalangle}$ are the interference FF that arise to leading order in $1/Q$ for the fragmentation of a current quark into two unpolarized hadrons inside the same jet, in the case under consideration a proton-pion pair. The different Dirac structures in the projections are related to different spin states of the fragmenting quark and lead to a nice probabilistic interpretation [1]: D_1 is the probability for an unpolarized quark to produce a pair of unpolarized hadrons; $G_1^\perp$ is the difference of probabilities for a longitudinally polarized quark with opposite chiralities to produce a pair of unpolarized hadrons; $H_1^{\sphericalangle}$ and $H_1^\perp$ both are differences of probabilities for a transversely polarized quark with opposite spins to produce a pair of unpolarized hadrons [1]. D_1 is chiral even and "T-even"; instead, $G_1^\perp, H_1^\perp, H_1^{\sphericalangle}$ are (naive) "T-odd". $G_1^\perp$ is also chiral even, while $H_1^\perp, H_1^{\sphericalangle}$ are chiral odd and, as pointed out in Ref. [1], represent the natural partner in a measurement of the transversity distribution h_1, in a a sort of "double" Collins effect [4].

III. SPECTATOR MODEL

In this section we extend the formalism of the so-called diquark spectator model [7–9] to calculation of the two-hadron interference FF, specializing it to the emission of a proton-pion pair.

The basic idea of the spectator model is to make a specific ansatz for the spectral decomposition of the quark correlator by replacing the sum over the complete set of intermediate states in Eq. (1) with an effective spectator state with a definite mass M_D and the quantum numbers of the diquark. Consequently, the correlator simplifies to

$$
\begin{aligned}
\Delta_{ij}(k; P_p, P_\pi) &= \sum\!\!\!\!\!\!\!\int_X \int \frac{d^4\zeta}{(2\pi)^4}\, e^{ik\cdot\zeta}\, \langle 0|\psi_i(\zeta)|\pi, p, X\rangle\langle X, p, \pi|\overline{\psi}_j(0)|0\rangle \\
&\approx \frac{\theta((k-P_h)^+)}{(2\pi)^3}\, \delta\left((k-P_h)^2 - M_D^2\right)\, \langle 0|\psi_i(0)|\pi, p, D\rangle\langle D, p, \pi|\overline{\psi}_j(0)|0\rangle \\
&\equiv \widetilde{\Delta}_{ij}(k; P_p, P_\pi)\, \delta(\tau_h - \sigma_h + M_h^2 - M_D^2)\,, \qquad (13)
\end{aligned}
$$

where in the last line use of the definition (4) for the kinematical invariants has been made. When inserting Eq. (13) into Eqs. (10)-(12), the additional δ function allows for a drastic simplification of the Dirac projections (9), namely

$$
\Delta^{[\Gamma]}(z_h, \xi, \vec{k}_T^{\,2}, \vec{R}_T^{\,2}, \vec{k}_T\cdot\vec{R}_T) = \left.\frac{\mathrm{Tr}[\widetilde{\Delta}\,\Gamma]}{8(1-z_h)P_h^-}\right|_{\tau_h=\tau_h(z_h,\vec{k}_T^{\,2})}\,, \qquad (14)
$$

with

$$
\tau_h(z_h, \vec{k}_T^{\,2}) = \frac{z_h}{1-z_h}\vec{k}_T^{\,2} + \frac{M_D^2}{1-z_h} + \frac{M_h^2}{z_h}\,. \qquad (15)
$$

With this hypothesis, the quark decay described in Fig. 1 is specialized to the set of diagrams shown in Figs. 3, 4 and their hermitean conjugates, where the interference, necessary to produce the "T-odd" FF, takes place between the channel for direct production from the quark q of the proton-pion pair (p,π) and the channel for the decay of the Roper resonance $\mathcal{R}$.

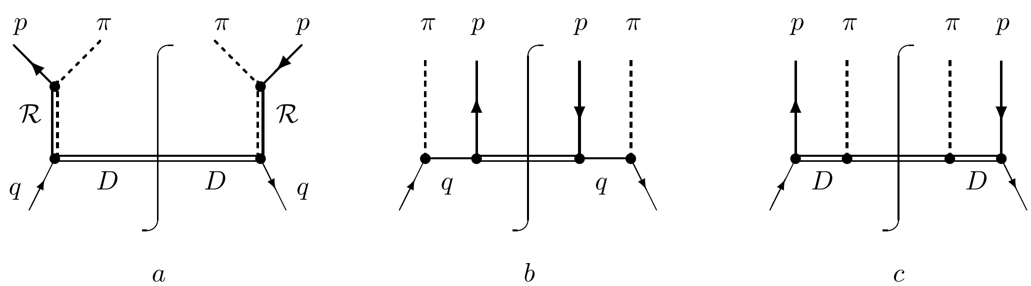


FIG. 3. Diagonal diagrams for quark q decay into a proton p and a pion π through a direct channel or a Roper resonance R.

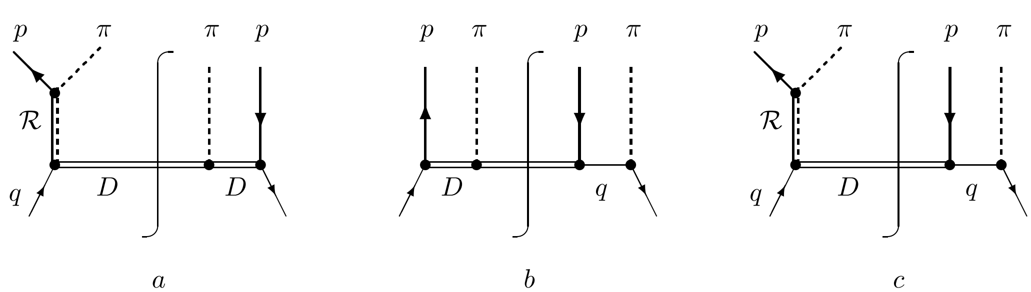


FIG. 4. Interference diagrams for the same process.

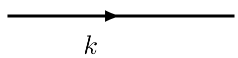
By modelling the various ingredients entering these diagrams, we can directly calculate the soft hadronic matrix elements of Eq. (13) and, consequently, make quantitative predictions for various Dirac projections and the related FF.

In the following, the most naive picture of the quark structure of the nucleon will be assumed, i.e. in the rest frame all quarks are in the $1/2^+$ orbitals and the diquark can be in a spin singlet state (scalar diquark, indicated by the label S) or in a spin triplet state (axial vector diquark, indicated by A). When generally referring to the diquark, the label D will be kept. We will further assume that the proton-pion pair has an invariant mass equal to the Roper resonance one, so that the diagrams containing an intermediate Roper $\mathcal{R}$ (Figs. 3a,4a,4c) will be dominant and all other channels (including diagrams 3b,3c, 4b and other resonances) will be neglected.

A. Propagators

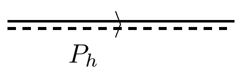
Here in the following, we list the propagators needed to compute the diagrams $3a$ and $4a$,$4c$.

- quark with momentum k



$$\left(\frac{i}{\not{k} - m}\right)_{ij}$$

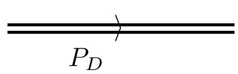
- Roper with momentum P_h



$$\left(\frac{i\,(\not{P}_h + M_{\mathcal{R}})}{P_h^2 - M_{\mathcal{R}}^2 + iM_{\mathcal{R}}\Gamma_{\mathcal{R}}}\right)_{ij}$$

as it is quoted by the Particle Data Group (PDG) [10] for the determination of the mass and the width of the Roper resonance.

- (scalar/axial vector) diquark with momentum $P_D = P_S/P_A$



$$\frac{i}{P_S^2 - M_S^2}$$

$$\frac{i}{P_A^2 - M_A^2}\left(-g_{\mu\nu} + \frac{P_{A\mu}\,P_{A\nu}}{M_A^2}\right)$$

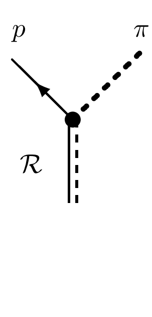
and the polarization sum for the axial vector diquark in the form $\sum_\lambda \epsilon_\mu^{*\,(\lambda)}\;\epsilon_\nu^{(\lambda)} \;= -g_{\mu\nu} + \dfrac{P_{A\mu}\,P_{A\nu}}{M_A^2}$.

B. Vertices

Here in the following, we list and discuss the interaction vertices Υ_{ij} needed to compute diagrams $3a$ and $4a$,$4c$. The normalization coefficients have proper mass dimensions such that $\int d^2\vec{k}_T \int d^2\vec{R}_T\; D_1(z_h, \xi, \vec{k}_T^{\,2}, \vec{R}_T^{\,2}, \vec{k}_T\cdot\vec{R}_T)$ be a dimensionless number to be interpreted as the probability for the hadron pair to carry a z_h fraction of the jet momentum and to share it in ξ and $1-\xi$ parts.

All vertices connecting hadron lines to parton lines will show a form factor that takes into account the composite structure of the hadron. A comment on the arguments of these form factors should be made at this point. Generally, a form factor can depend on all invariants which can be built from the momenta of the attached lines. In analogy with previous works on the spectator model [8,9] we have chosen a form depending on one invariant only, generically denoted κ^2 in the following. It is chosen as the virtuality of the external quark line τ_h whenever the corresponding vertex is attached to it, or as the virtuality of the intermediate diquark (quark) propagator for the $A\pi A$-vertex (qDp-vertex) in Fig. 4 diagram a (c). With these choices of arguments of the form factors, all virtual parton lines in the calculation receive an appropriate suppression preventing them from being far off-shell. The expressions of form factors are such that the net result is an asymptotic behaviour in agreement with expectations based on dimensional counting rules.

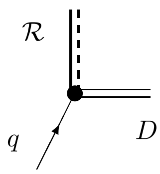
- Roper-proton-pion ($\mathcal{R}p\pi$)



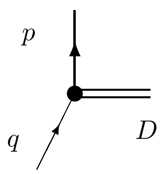
$$\Upsilon_{ij}^{\mathcal{R}p\pi} = f_{\mathcal{R}p\pi}\;[\gamma_5]_{ij}\;\equiv g\;[\gamma_5]_{ij}\;,$$

where $g^2/4\pi = 14.3$ is the strong coupling constant of the πNN pseudoscalar interaction. Calculation of the Roper-proton-pion coupling $f_{\mathcal{R}p\pi}$ by means of two independent ways, namely the Roper decay and the elastic $p+\pi \to p+\pi$ scattering through a virtual channel with the same mass and width of the Roper, produces a decay width and an empirical channel strength, respectively, that depend on $f_{\mathcal{R}p\pi}$. Comparison with experimental data allows $f_{\mathcal{R}p\pi}$ to be compatible with g within the large experimental error bars. Hence, in the following in all vertices involving a Roper we will keep the same form and strength as in the ones involving a proton, also because, being the quark content the same, the asymptotic behaviour of the form factors should be the same.

- quark-diquark-(Roper/proton) ($qD\mathcal{R}/qDp$)



$$\Upsilon_{ij}^{qS\mathcal{R}/qSp} = f_{(qS\mathcal{R}/qSp)}(\kappa^2)\ \mathbf{1}_{ij} \equiv N_{qS}\ \frac{\kappa^2 - m^2}{|\kappa^2 - \Lambda^2|^\alpha}\ \mathbf{1}_{ij}$$



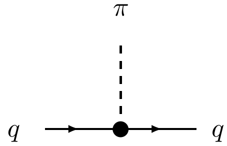
$$\Upsilon_{ij}^{qA\mathcal{R}/qAp,\ \mu} = f_{(qA\mathcal{R}/qAp)}(\kappa^2)\ [\gamma_5\gamma^\mu]_{ij} \equiv \frac{N_{qA}}{\sqrt{3}}\ \frac{\kappa^2 - m^2}{|\kappa^2 - \Lambda^2|^\alpha}\ [\gamma_5\gamma^\mu]_{ij}\ ,$$

where m is the quark mass and $\Lambda = 0.5$ GeV is a cut-off parameter that excludes large virtualities of the quark. This choice of the form factor has the advantage of killing the pole of the quark propagator [8,9]. As remarked previously the argument of the (qDp) form factor is $\kappa^2 = \tau_h$ in diagram $4(a)$ and $\kappa^2 = (k - P_\pi)^2$ in diagram $4(c)$. The power law α is determined consistently with the quark counting rule that drives the asymptotic behaviour of the FF at large z_h [11], i.e.

$$(1 - z_h)^{2\alpha - 1} \equiv (1 - z_h)^{-3+2r+2|\lambda|}\ , \tag{16}$$

where r is the number of constituent quarks in the considered hadron, and λ is the difference between the quark and hadron helicities. Thus, in this case $\alpha = 2$. To determine the form of the vertex involving the axial vector diquark we took inspiration from considerations which are valid for on-shell particles, only. In that case, the most general form for the vertex involving the axial vector diquark is $\gamma_5\gamma^\mu + \gamma_5(ap^\mu + bk^\mu)$, with a, b arbitrary parameters. The second piece can be rewritten using the difference $p^\mu - k^\mu = P_A^\mu$ and the sum $(p + k)^\mu$ of the momenta involved. The former is ruled out because in configuration space it is equivalent to taking the 4-divergence of the spin-1 diquark field, $\partial_\mu \psi_A^\mu$: the Lorentz condition $\partial_\mu \psi_A^\mu = 0$ is usually taken to grant a field energy which is positive defined. The latter can be transformed, using a Gordon-like decomposition, into the linear combination of $\gamma_5\gamma^\mu$ (to be reabsorbed into the initial first piece) and $\gamma_5\sigma^{\mu\nu}P_{A\nu}$ which, contrary to the axial current, is even under G-parity transformations and, therefore, is forbidden [12]. Hence, the general structure $\gamma_5\gamma^\mu$ is assumed for this vertex. The overall normalizations $N_{qS} = 7.92$ GeV^2 and $N_{qA} = 11.557$ GeV^2 are fixed by computing the second moment of $D_1(z_h)$ and comparing it with available data; their values, as well as the Λ parameter, are taken directly from Ref. [9].

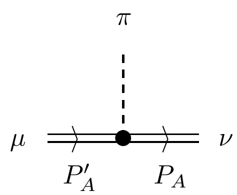
- quark-pion-quark ($q\pi q$)



$$\Upsilon_{ij}^{q\pi q} = f_{q\pi q}(\kappa^2)\ [\gamma_5]_{ij} \equiv N_{q\pi}\ \frac{\kappa^2 - m^2}{|\kappa^2 - \Lambda_\pi^2|^\alpha}\ [\gamma_5]_{ij}\ ,$$

where the new parameters $\Lambda_\pi = 0.4$ GeV, $N_{q\pi} = 2.564$ GeV have the same meaning and role as before and are determined in a similar manner [9]. In this case, Eq. (16) gives $\alpha = 3/2$.

- (axial) diquark-pion-(axial) diquark ($A\pi A$)



$$
\begin{aligned}
\Upsilon^{A\pi A}_{\mu\nu} &= i\, f_{A\pi A}(\kappa^2)\ \epsilon_{\mu\nu\rho\sigma}\, P'^{\rho}_A\, P^{\sigma}_A \\
&= -i\ N_{\pi A}\ \frac{\kappa^2 - M_A^2}{|\kappa^2 - \Lambda_\pi^2|^\alpha}\ \epsilon_{\mu\nu\rho\sigma}\, P'^{\rho}_A\, P^{\sigma}_A\ .
\end{aligned}
$$

Only the axial diquarks are here considered because two scalar diquarks cannot couple to a pseudoscalar particle. The overall negative sign, together with the Dirac structure and the strength $N_{\pi A}$, are justified the Appendix. The parameters Λ_π and α are determined as before. In this case, Eq. (16) gives $\alpha = 2$.

C. Calculation of fragmentation functions

Using the results of the previous Sections III A, III B we can calculate the soft hadronic matrix elements of Eq. (13) corresponding to the diagrams $3a$,$4a$,$4c$ and their hermitean conjugates; consequently, all the Dirac projections needed to determine the FF $D_1, G_1^\perp, H_1^\perp, H_1^{\triangleleft}$ can be computed according to Eq. (14) and (10)-(12).

In agreement with theoretical expectations [1], the "T-even" D_1 gets the dominant contribution from the diagonal diagram $3a$, while the naive "T-odd" $G_1^\perp$, $H_1^\perp$, $H_1^{\triangleleft}$ are entirely driven by the interference diagrams $4a$,$4c$, thus confirming that in this case there is a close relation between the "T-odd" structure of the cross section to leading order and the FSI produced by the interference of the two channels for proton-pion production. Therefore, in the following, we will concentrate only on the "T-odd" FF.

Before addressing the results, some remarks must be added about the flavor decomposition of FF, in particular about the different role played by the scalar and the axial diquarks. In fact, as demanded by Pauli principle, if the total spin-flavor wave function of a baryon must be symmetric, the scalar diquark must be in a flavor singlet state while the axial diquark in a flavor triplet state. This leads, e.g., to the known $SU(4)$ structure of the proton wave function [9] and to the ratio 3:1 between the contributions of the scalar and axial diquarks, respectively, for the fragmentation of an u quark. We assume that the spin-flavor content of the Roper wave function is similar to the proton one. Therefore, Eqs. (11), (12) for the fragmentation of an u quark into a proton and a pion, $u \to p + \pi$, get contributions from

$$
\begin{aligned}
\Delta^{[\gamma^-\gamma_5]}(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T\cdot\vec{R}_T) &\equiv \frac{\epsilon_T^{ij}\, R_{Ti}\, k_{Tj}}{M_1\, M_2}\ G_1^\perp = \left.\frac{\mathrm{Tr}[\widetilde{\Delta}\,\gamma^-\gamma_5]}{8(1-z_h)P_h^-}\right|_{\tau_h=\tau_h(z_h,\vec{k}_T^2)} \\
&= \frac{1}{8(1-z_h)P_h^-}\ \mathrm{Tr}\left.\left[\left(\frac{1}{2}\widetilde{\Delta}_{IaA} + \frac{3}{2}\widetilde{\Delta}_{IcS} + \frac{1}{2}\widetilde{\Delta}_{IcA}\right)\gamma^-\gamma_5\right]\right|_{\tau_h=\tau_h(z_h,\vec{k}_T^2)}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\Delta^{[i\sigma^{i-}\gamma_5]}(z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T\cdot\vec{R}_T) &\equiv \frac{\epsilon_T^{ij}\, R_{Tj}}{M_1 + M_2}\ H_1^{\triangleleft} + \frac{\epsilon_T^{ij}\, k_{Tj}}{M_1+M_2}\ H_1^\perp = \left.\frac{\mathrm{Tr}[\widetilde{\Delta}\, i\sigma^{i-}\gamma_5]}{8(1-z_h)P_h^-}\right|_{\tau_h=\tau_h(z_h,\vec{k}_T^2)} \\
&= \frac{1}{8(1-z_h)P_h^-}\ \mathrm{Tr}\left.\left[\left(\frac{1}{2}\widetilde{\Delta}_{IaA} + \frac{3}{2}\widetilde{\Delta}_{IcS} + \frac{1}{2}\widetilde{\Delta}_{IcA}\right) i\sigma^{i-}\gamma_5\right]\right|_{\tau_h=\tau_h(z_h,\vec{k}_T^2)}\ ,
\end{aligned}
\tag{18}
$$

where $\widetilde{\Delta}_{I(a/c)(S/A)}$ refers to the soft matrix element of diagram $4(a/c)$ involving the (scalar/axial) diquark, respectively. Their relative weights are given by the proper Clebsch-Gordan coefficients for the combination of isospins according to the above mentioned $SU(4)$ symmetry. The latter is responsible also for the equivalence $(u \to p) \Leftrightarrow (d \to n)$, therefore the previous equation can usefully be adopted to describe the d quark fragmentation into a neutron and a pion. On similar grounds, the fragmentation $u \to n + \pi$, or equivalently $d \to p + \pi$, gets contribution only from the diagrams involving the axial diquark only [9].

By working out the matrix elements included in Eqs. (17), (18), we get the explicit expressions for the naive "T-odd" FF for the process $u \to p + \pi$ at leading order in the context of the diquark spectator model, i.e.

$$
G_1^{\perp\ u\to p+\pi}(z_h, \xi, \tau_h, M_h^2, \sigma_d) = \frac{2}{(2\pi)^3}\ \frac{\Gamma_{\mathcal{R}} M_{\mathcal{R}}}{(M_h^2 - M_{\mathcal{R}}^2)^2 + M_{\mathcal{R}}^2\Gamma_{\mathcal{R}}^2}\ \frac{M_p M_\pi}{1-z_h}\ \Bigg\{
$$

$$
\begin{aligned}
&+\frac{1}{2}\,\frac{2\,f_{A\pi A}\,f_{\mathcal{R}p\pi}\,f_{qAp}\,f_{qA\mathcal{R}}}{3(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_p^2-M_A^2-\sigma_d\right)}\,\left\{m\left(M_h^2-2M_pM_{\mathcal{R}}+M_p^2-M_\pi^2\right)\right\}\\
&+\frac{3}{2}\,\frac{f_{q\pi q}\,f_{qSp}\,f_{\mathcal{R}p\pi}\,f_{qS\mathcal{R}}}{(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_\pi^2+M_S^2-2m^2+\sigma_d\right)}\,\left\{\tau_h-m^2+2\,m(M_{\mathcal{R}}-M_p)\right\}\\
&-\frac{1}{2}\,\frac{f_{q\pi q}\,f_{qAp}\,f_{\mathcal{R}p\pi}\,f_{qA\mathcal{R}}}{6M_A^2\,(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_\pi^2+M_A^2-2m^2+\sigma_d\right)}\,\Big\{-\left(\tau_h-m^2\right)^2\\
&\quad+\left(\tau_h+m^2+2mM_{\mathcal{R}}\right)\left[M_h^2+2(M_p^2-M_\pi^2)-M_A^2+m^2+2m(2M_p-M_{\mathcal{R}})-\sigma_d\right]\\
&\quad-4\,m\,\left[M_p(M_h^2-2M_A^2+m^2)+mM_h^2+(2M_A^2+mM_{\mathcal{R}})(2M_p-M_{\mathcal{R}})\right]\Big\}\quad\Bigg\}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&H_1^{\sphericalangle\,u\to p+\pi}(z_h,\xi,\tau_h;M_h^2,\sigma_d)=\frac{2}{(2\pi)^3}\,\frac{\Gamma_{\mathcal{R}}M_{\mathcal{R}}}{(M_h^2-M_{\mathcal{R}}^2)^2+M_{\mathcal{R}}^2\Gamma_{\mathcal{R}}^2}\,\frac{M_p+M_\pi}{z_h\,(1-z_h)}\,\Bigg\{\\
&+\frac{1}{2}\,\frac{f_{A\pi A}\,f_{\mathcal{R}p\pi}\,f_{qAp}\,f_{qA\mathcal{R}}\,\left(M_h^2+M_p^2-M_\pi^2-2M_pM_{\mathcal{R}}\right)}{3(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_p^2-M_A^2-\sigma_d\right)}\left\{M_h^2-M_A^2-z_h\tau_h+m^2(1-z_h)\right\}\\
&+\frac{3}{2}\,\frac{f_{q\pi q}\,f_{qSp}\,f_{\mathcal{R}p\pi}\,f_{qS\mathcal{R}}}{2(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_\pi^2+M_S^2-2m^2+\sigma_d\right)}\\
&\quad\times\left\{2[M_h^2-M_S^2+m^2(1-2z_h)](M_{\mathcal{R}}-M_p)+2(\tau_h-m^2)[(1-z_h)(M_{\mathcal{R}}-M_p)-z_hm-M_{\mathcal{R}}]\right\}\\
&-\frac{1}{2}\,\frac{f_{q\pi q}\,f_{qAp}\,f_{\mathcal{R}p\pi}\,f_{qA\mathcal{R}}}{6M_A^2\,(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_\pi^2+M_A^2-2m^2+\sigma_d\right)}\\
&\quad\times\Big\{\tau_h^2\,(-2z_hM_p+z_hM_{\mathcal{R}}-m)+M_h^2[M_h^2+(1-z_h)m^2](M_{\mathcal{R}}-2M_p-m)-m^3M_h^2(1-z_h)\\
&\qquad+(M_p^2-M_\pi^2)[2(1-z_h)m^2M_{\mathcal{R}}+2(M_h^2-M_A^2)(M_{\mathcal{R}}+m)]+M_A^4(4M_p-3M_{\mathcal{R}}+m)\\
&\qquad+M_A^2\left[-2M_h^2(M_p-M_{\mathcal{R}})+m^2[-2(1-2z_h)M_p+3(1-z_h)M_{\mathcal{R}}-2m]\right]\\
&\qquad+\tau_h\Big[2(M_p^2-M_\pi^2)[m(1-2z_h)-z_hM_{\mathcal{R}}]+\sigma_d[-m(1-2z_h)+z_hM_{\mathcal{R}}]+M_h^2(1+z_h)(2M_p-M_{\mathcal{R}})\\
&\qquad\quad+M_A^2[M_{\mathcal{R}}(1-3z_h)-4M_p(1-z_h)+2z_hm]+m[2m^2(1-z_h)+m(1-z_h)(2M_p-M_{\mathcal{R}})+2M_h^2]\Big]\\
&\qquad+\sigma_d[(M_A^2-M_h^2)(M_{\mathcal{R}}+m)-m^2M_{\mathcal{R}}(1-z_h)]\Big\}\quad\Bigg\}
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&H_1^{\perp\,u\to p+\pi}(z_h,\xi,\tau_h,M_h^2,\sigma_d)=\frac{2}{(2\pi)^3}\,\frac{\Gamma_{\mathcal{R}}M_{\mathcal{R}}}{(M_h^2-M_{\mathcal{R}}^2)^2+M_{\mathcal{R}}^2\Gamma_{\mathcal{R}}^2}\,\frac{M_p+M_\pi}{1-z_h}\,\Bigg\{\\
&+\frac{1}{2}\,\frac{f_{A\pi A}\,f_{\mathcal{R}p\pi}\,f_{qAp}\,f_{qA\mathcal{R}}\,\left(M_h^2+M_p^2-M_\pi^2-2M_pM_{\mathcal{R}}\right)}{6(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_p^2-M_A^2-\sigma_d\right)}\\
&\quad\times\left\{-(\tau_h-m^2)+\sigma_d+(M_h^2-M_A^2+m^2)(1-2\xi)\right\}\\
&+\frac{3}{2}\,\frac{f_{q\pi q}\,f_{qSp}\,f_{\mathcal{R}p\pi}\,f_{qS\mathcal{R}}}{2(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_\pi^2+M_S^2-2m^2+\sigma_d\right)}\\
&\quad\times\left\{(1-2\xi)\left(M_h^2-M_S^2+m^2\right)(M_{\mathcal{R}}-M_p)+\left(\sigma_d+\tau_h-m^2\right)(M_{\mathcal{R}}-M_p)-2(1-\xi)\left(\tau_h-m^2\right)M_p\right\}
\end{aligned}
$$

$$
-\frac{1}{2}\,\frac{f_{q\pi q}\; f_{qAp}\; f_{\mathcal{R}p\pi}\; f_{qA\mathcal{R}}}{12M_A^2\,(\tau_h-m^2)^2\,\left(\tau_h-M_h^2+2M_\pi^2+M_A^2-2m^2+\sigma_d\right)}
$$

$$
\begin{aligned}
\times\Big\{ & -\tau_h^2\left[m(1-2\xi)-(2M_p-M_\mathcal{R})\right]-\sigma_d^2(m+M_\mathcal{R})+2M_A^2m^2\left[2M_p(1+\xi)-3\xi M_\mathcal{R}-m(1-2\xi)\right]\\
& +(M_p^2-M_\pi^2)\left[2(M_h^2-M_A^2)(m+M_\mathcal{R})(1-2\xi)-4m^2(\xi M_\mathcal{R}+m)\right]\\
& -(M_h^4-M_A^4)(2M_p-M_\mathcal{R}+m)(1-2\xi)+2M_A^2(M_A^2-M_h^2)(M_p-M_\mathcal{R})(1-2\xi)\\
& +2M_h^2m^2\left[\xi(2M_p-M_\mathcal{R})-m(1-2\xi)\right]\\
& +\tau_h\Big[2\sigma_d[M_p-M_\mathcal{R}-m(1-\xi)]+2M_A^2[M_\mathcal{R}(2-\xi)-M_p(5-4\xi)]+2\xi M_\mathcal{R}(m^2+M_h^2)\\
& \qquad +(M_p^2-M_\pi^2)[m(6-4\xi)+2M_\mathcal{R}]+m[2(M_h^2+m^2)(1-2\xi)-4m\xi M_p]-4\xi M_pM_h^2\Big]\\
& -2\sigma_d\Big[M_A^2[2(M_p-M_\mathcal{R})+\xi(M_\mathcal{R}+m)]+m[(1-\xi)M_h^2-m(m+\xi M_\mathcal{R})]-M_h^2(\xi M_\mathcal{R}-M_p)\\
& \qquad -(M_\mathcal{R}+m)(M_p^2-M_\pi^2)\Big]\Big\}\quad\Big\}\,,
\end{aligned}
\tag{21}
$$

where M_p, M_π, M_S, M_A are the proton, pion, scalar and axial diquark masses, respectively, and $M_\mathcal{R}, \Gamma_\mathcal{R}$ are the mass and the experimental width of the Roper resonance. The explicit dependence of FF upon the invariants described in Eq. (4), rather than on the variables shown in Eqs. (17), (18), is due just to convenience of making the formula the most easily readable. In the following, plots will be shown of FF considered again as functions of $z_h, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T\cdot\vec{R}_T$, which allow for a better understanding of the underlying physics, as stressed in Sec. II.

IV. NUMERICAL RESULTS

Plots of FF in Eqs. (19)-(21) are shown for the parameter values (in GeV)

$$
m=0.36\ ,\ M_S=0.6\ ,\ M_A=0.8\ ,\ M_p=0.9383\ ,\ M_\pi=0.1396\ . \tag{22}
$$

As already anticipated in Sec. III, the proton-pion invariant mass will be taken equal to the Roper resonance mass, $M_h = M_\mathcal{R} = 1.44$ GeV, so that diagrams of Figs. 3*b*, 3*c*, 4*b* are then negligible. The Roper resonance width is $\Gamma_\mathcal{R} = 0.35$ GeV [10].

For the purpose of displaying our results we will choose a special kinematics where $\vec{k}_T\cdot\vec{R}_T=0$, namely where the hadron-pair plane is perpendicular to the plane containing the jet axis and the direction of the hadron pair $\vec{P}_h$ (see Fig. 2, where the indicated angle α takes the value 90^o). Consequently, it turns out from Eq. (4b) that $\sigma_d=\sigma_d(z_h,\xi,\vec{k}_T^2,\vec{R}_T^2)$. Moreover, from Eq. (15) also the invariant τ_h depends on $z_h,\vec{k}_T^2$ and from Eq. (5) we have $\vec{R}_T^2(\xi)$. Therefore, at fixed invariant mass $M_h=M_\mathcal{R}$ and $\vec{k}_T\cdot\vec{R}_T=0$ the FF are actual functions of $z_h,\xi,\vec{k}_T^2$ only. Equation (5) further constrains the variable ξ, because from the positivity of $\vec{R}_T^2(\xi)$ we have

$$
M_h^2 \geq \frac{M_p^2}{\xi}+\frac{M_\pi^2}{1-\xi}\ , \tag{23}
$$

and, for the above fixed values of the masses, $0.43\leq\xi\leq 0.98$.

In Fig. 5 the interference FF $H_1^{\sphericalangle\, u\to p+\pi}$, $H_1^{\perp\, u\to p+\pi}$, $G_1^{\perp\, u\to p+\pi}$ are shown (from top to bottom, respectively) as functions of z_h, $\vec{k}_T^2$ at $\xi=0.7$. Similar results are obtained for different values of ξ in the allowed range. The maximum sensitivity to the fragmentation mechanism is concentrated around the kinematical range where the pair takes roughly 80% of the jet energy ($z_h\sim 0.8$) and has a small transverse momentum with respect to the jet axis ($\vec{k}_T^2\lesssim 0.4$ GeV^2; we recall that $G_1^\perp$, $H_1^{\sphericalangle}$, $H_1^\perp$ are defined as the probability difference for the unpolarized hadron pair to be generated from the fragmentation of a quark with parallel or transverse polarization with respect to its longitudinal momentum).

It is worth noting also from the plot scales that $G_1^\perp$ is roughly smaller by one order of magnitude with respect to the other pair of FF.

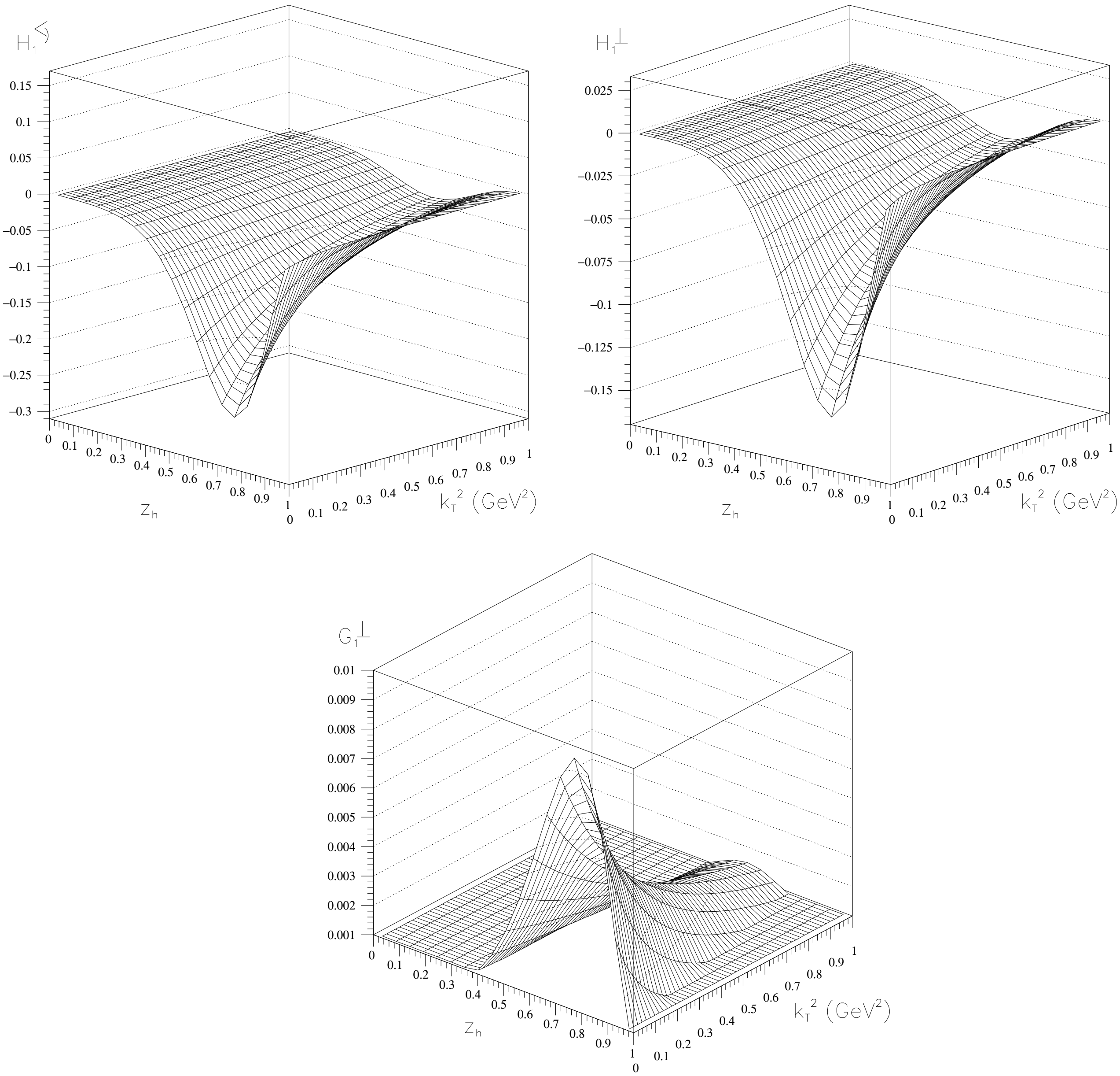


FIG. 5. $H_1^{\sphericalangle}(z_h, \vec{k}_T^2)$, $H_1^{\perp}(z_h, \vec{k}_T^2)$, $G_1^{\perp}(z_h, \vec{k}_T^2)$ at $\xi = 0.7$ (from top to bottom) for the fragmentation of a quark u into a proton p and a pion π. Kinematics is chosen such that the invariant mass of the pair is equal to the Roper resonance and $\vec{k}_T \cdot \vec{R}_T = 0$ (see Fig. 2).

By "cutting" the 3-d surfaces of Fig. 5 at constant values of z_h in the interesting range, e.g. for $z_h \geq 0.6$, one can get the trend of the dependence on transverse quark momenta in the non-perturbative low $\vec{k}_T^2$ region. In Fig. 6, all this is shown for $G_1^{\perp}$ in the same conditions and kinematics as in Fig. 5 (i.e., for $\xi = 0.7$ and $\vec{k}_T \cdot \vec{R}_T = 0$) and for $z_h = 0.6, 0.7, 0.8, 0.9$. It is evident that for increasing z_h, the fragmentation function gets concentrated at lower $\vec{k}_T^2$ and have an increasingly less important tail. This result can be generalized also to the other FF. In other words, the more the hadron pair is leading, i.e. it takes most of the jet energy, the more the FF are concentrated around the jet axis with smaller transverse momentum.

The asymptotic behaviour of the FF for very large $\vec{k}_T^2$ is dictated by the form factors at each vertex. It is instructive to look at the quark transverse momentum dependence for the contributions from each diagram separately. From a simple power counting in propagators and vertices (including form factors) in the diagrams,

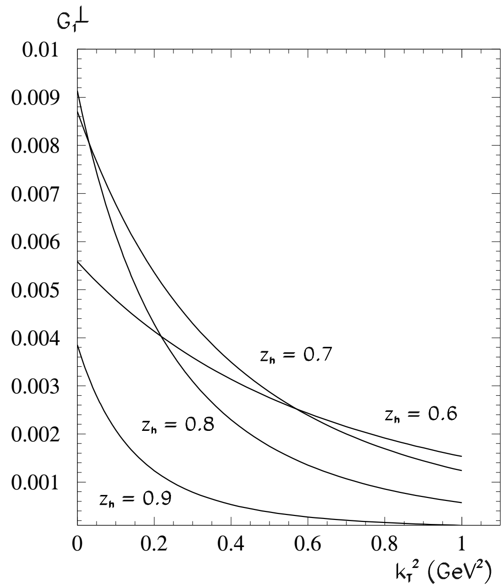


FIG. 6. $G_1^\perp(\vec{k}_T^2)$ at $\xi = 0.7$ and $z_h = 0.6, 0.7, 0.8, 0.9$ for the fragmentation of a quark u into a proton p and a pion π. Kinematics as in previous Fig. 5.

one can obtain the lower limit $\bar{n}$ in the $1/(\vec{k}_T^2)^n$ asymptotic decrease. Cancellations of leading power terms in the numerators can result in an even faster decrease of the actual results for the FF, i.e. a higher power in the asymptotic $1/(\vec{k}_T^2)^n$ behavior. In Table I we compare $\bar{n}$ from each diagram with the actual asymptotic behavior of their contributions to the FF.

	$\widetilde{\Delta}(3aS)$	$\widetilde{\Delta}(3aA)$	$\widetilde{\Delta}(4aA)$	$\widetilde{\Delta}(4cS)$	$\widetilde{\Delta}(4cA)$
$\bar{n}$	3	2	4	4	3
$n(G_1^\perp)$	-	-	6	4.5	3.5
$n(H_1^\perp)$	-	-	5	4.5	3.5
$n(H_1^\sphericalangle)$	-	-	5	4.5	3.5

TABLE I. The lower limit $\bar{n}$ in the $1/(\vec{k}_T^2)^{\bar{n}}$ asymptotic decrease as obtained from a power counting. It is compared with the power actually found in the FF. The power law for the contributions from each diagram are shown separately, where $\widetilde{\Delta}((3/4)(a/c)(S/A))$ refers to the diagram (a/c) in Fig. 3/4 involving the (scalar/axial) diquark.

As already found in a different context [13], the lower limit $\bar{n}$ is found to be smaller for the dominant "pure" channel related to the diagonal diagram $3a$, i.e. it shows the slowest asymptotic fall-off in $\vec{k}_T^2$. This agrees with the expectation from dimensional counting rules which link the asymptotic behavior to the number of outgoing partons in the lowest Fock state: 3 quarks in the Roper compared to 5 (anti-) quarks in the proton-pion pair (the subsequent Roper decay takes place spontaneously and does not affect the parton counting in this argument).

We have checked our results for stability under different choices for the quark mass parameter, and found a modest dependence. Even a drastic choice like setting the quark mass to zero results in less then 20% changes of the FF functions (except for the extreme end-points of the kinematical regions where the results with zero and non-zero m_q both vanish, and quoting of a relative size does not make sense).

V. SUMMARY

In this paper we have calculated the interference fragmentation functions (FF) that arise from the distribution of two hadrons produced in the same jet in the current fragmentation region of a hard process, e.g., in two-hadron inclusive lepton-nucleon scattering.

Naive "T-odd" FF generally arise because the existence of Final State Interactions (FSI) prevents constraints from time-reversal invariance to be applied. This class of FF is interesting because it allows for a direct investigation of mechanisms for residual interactions inside jets and can contain the chiral-odd partners needed to isolate the presently unknown quark transversity distribution h_1.

The presence of FSI allows that in the fragmentation process there are at least two interfering competing channels. However, it has been shown [1] that this is not enough to generate "T-odd" FF. A realistic microscopic description of the vertices involved in the different channels is required which naturally selects two-hadron emission inside the same jet as the most convenient scenario.

To leading order, four FF arise, among which three are naive "T-odd" and are related to the fragmenting quark being polarized longitudinally ($G_1^\perp$) or transversely ($H_1^\perp, H_1^{\sphericalangle}$). The latter two ones are, moreover, chiral odd and can be identified as the partners to isolate h_1. In fact, asymmetry measurements in two-hadron inclusive DIS on a transversely polarized nucleon target induced by an unpolarized beam can be envisaged [1], where $H_1^{\sphericalangle}$, $H_1^\perp$ enter the cross section in convolutions with h_1. In particular, $H_1^{\sphericalangle}$ survives the integration over the transverse momentum of the fragmenting quark and can be deconvoluted from the transversity distribution.

Calculations of such FF have been shown extending the diquark spectator model of Refs. [7–9] and specializing it to the case of the hadron pair being a proton and a pion produced either directly or through the Roper resonance. Therefore, this is the first example of an explicit, complete and detailed model calculation of "T-odd" FF.

A well-known problem of this kind of approach is that it does not provide a scale dependence. In principle, it is a "low-scale model", since it involves simple valence quark degrees of freedom. In Ref. [9] a comparison with parametrizations at "low hadronic scale" and available experimental data has been attempted and a reasonable qualitative agreement has been obtained for the known distribution functions and $D_1(z_h)$. These findings give a good estimate for the values of the model parameters, that here are also adopted to calculate the other unknown fragmentation functions. Moreover, they give an indication of the level of accuracy that can be reached disregarding sea-quarks, gluons and evolution.

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APPENDIX:

In this Appendix we give a justification for the Dirac structure, sign and strength of the coupling adopted in Sec. III for the (axial) diquark-pion-(axial) diquark vertex ($A\pi A$). In the following, we refer to the same notations and conventions of Ref. [14].

The axial diquark is represented by a spin-1 field ψ_μ that satisfies the equation of motion subject to the Lorentz condition,

$$p^2\psi_\mu = M_A^2\psi_\mu \quad ; \quad p^\mu\psi_\mu = 0 \; , \tag{A1}$$

while the field tensor is defined as $F_{\mu\nu} = \partial_\mu\psi_\nu - \partial_\nu\psi_\mu$. The conserved vector current is given by

$$V^\mu = i\left((F'^{\mu\nu})^*\psi_\nu - F^{\mu\nu}\psi_\nu'^*\right) \; , \tag{A2}$$

where $F'_{\mu\nu} = \partial_\mu\psi'_\nu - \partial_\nu\psi'_\mu$ and $\psi^{(')}$ refers to the incoming (outgoing) spin-1 field in the vertex. Analogously, we define the diquark axial current

$$A_0^\mu = f_{A\pi A}\;\epsilon^{\mu\alpha\beta\gamma}\left(F'^*_{\beta\gamma}\psi_\alpha - F_{\beta\gamma}\psi'^*_\alpha\right) \;=\; 2if_{A\pi A}\;\epsilon^{\mu\alpha\beta\gamma}\left(\psi'^*_\alpha\;p_\beta\;\psi_\gamma - \psi'^*_\gamma\;p'_\beta\;\psi_\alpha\right) \; , \tag{A3}$$

where p' refers to the momentum of the outgoing diquark. The $f_{A\pi A}$ is the weak axial-diquark coupling used in Sec. III B, whose sign, however, is not yet determined, and the definition of $F^{\mu\nu}$ in momentum space has been

used. The current A_0^μ does not fulfil the PCAC hypothesis. Analogously to the weak neutron decay, we redefine the axial current introducing a pole term to restore the PCAC hypothesis (or CAC, in case of vanishing pion mass m_π), i.e.

$$A^\mu = A_0^\mu + A_1^\mu \equiv (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2 - m_\pi^2}) A_{0\nu} \ , \tag{A4}$$

where q^μ is the momentum transferred from the vertex. The pole term contains the diquark-pion-diquark ($A\pi A$) vertex of interest. Its explicit expression gives the Dirac structure of this vertex:

$$\begin{aligned} A_1^\mu &= -\frac{q^\mu q^\nu}{q^2 - m_\pi^2} A_{0\nu} \ = \ -\frac{q^\mu \, (p^\nu - p'^\nu)}{q^2 - m_\pi^2} \, 2if_{A\pi A} \ \epsilon_{\nu\alpha\beta\gamma} (\psi'^{*\alpha} p^\beta \psi^\gamma - \psi'^{*\gamma} p'^\beta \psi^\alpha) \\ &= 4 \frac{q^\mu}{q^2 - m_\pi^2} \ \psi'^{*\alpha} \ if_{A\pi A} \ \epsilon_{\nu\beta\alpha\gamma} \ p^\nu p'^\beta \ \psi^\gamma \ \equiv \ 4 \frac{q^\mu}{q^2 - m_\pi^2} \ \psi'^{*\alpha} \ \Upsilon_{\alpha\gamma}^{A\pi A} \ \psi^\gamma \ . \end{aligned} \tag{A5}$$

In order to determine whether $f_{A\pi A}$ is positive or negative, we need to explicitly calculate a diagram involving A_0^μ, as in a hypothetical weak neutron decay $n \to p + e^- + \nu_e$ via an axial diquark depicted in Fig. 7. Since the total neutron current has the $V - A \Leftrightarrow \gamma^\mu - \gamma^\mu \gamma_5 \equiv \gamma^\mu + \gamma_5 \gamma^\mu$ structure, our strategy will be to calculate A_0^μ from the diagram of Fig. 7 and then project out the $\gamma_5 \gamma^\mu$ part: the sign of the result will determine the sign of $f_{A\pi A}$ with respect to the vector current V^μ.

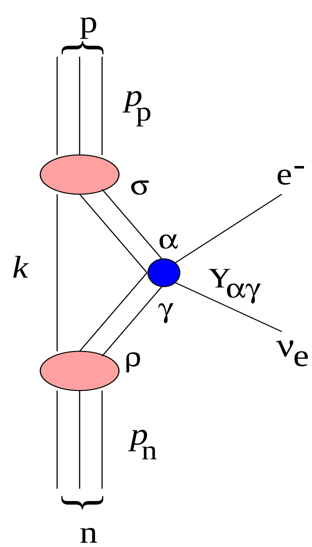

FIG. 7. Weak decay of a neutron with momentum p_n into a proton with momentum p_p via an axial diquark.

To calculate the diagram of Fig. 7 we use the rules described in Sec. III for the qAp vertex, the $A\pi A$ vertex and the axial diquark propagator. The net result is a current G^μ given by

$$\begin{aligned} G^\mu = \int d^4k \ & \frac{1}{k^2 - m^2} \ \frac{1}{(p_p - k)^2 - M_A^2} \ \frac{1}{(p_n - k)^2 - M_A^2} \\ & \times \ 2if_{A\pi A} \ f_{qAp}^2 \ \gamma^\sigma \ \not{k} \gamma^\rho \ \epsilon^{\mu\alpha\beta\gamma} \left[(p_n - k)_\beta \ g_{\alpha\sigma} g_{\rho\gamma} - (p_p - k)_\beta \ g_{\gamma\sigma} g_{\rho\alpha} \right] \ . \end{aligned} \tag{A6}$$

Moreover, the Dirac-structure decomposition of a matrix M contains the $\gamma_5 \gamma^\mu$ projection as

$$M = \ldots - \frac{1}{4} \mathrm{Tr}[\gamma_5 \gamma^\mu M] \ \gamma_5 \gamma^\mu \ldots \ . \tag{A7}$$

Therefore, the $\gamma_5 \gamma^\nu$ projection of G^μ will be

$$\begin{aligned} -\tfrac{1}{4} \mathrm{Tr}[\gamma_5 \gamma^\nu G^\mu] = \int d^4k \ & \frac{1}{k^2 - m^2} \ \frac{1}{(p_p - k)^2 - M_A^2} \ \frac{1}{(p_n - k)^2 - M_A^2} \\ & \times \ \left\{ -4 f_{A\pi A} \ f_{qAp}^2 \ \left[k^\mu \ (p_n + p_p - 2k)^\nu - k \cdot (p_n + p_p - 2k) \ g^{\mu\nu} \right] \right\} \end{aligned} \ . \tag{A8}$$

To determine the sign of the r.h.s. of Eq. (A8), let us consider the component $\mu = \nu = 0$ only. Noting that strong off-shellness of the particles is suppressed by the form factors, propagators in Eq. (A8) are dominated by their positive energy poles. Writing the quark energy as $k^0 = m+\epsilon$ with $0 < \epsilon \ll m$, and taking $m_p = m_n \sim 3m$ and $M_A \sim 2m$, we finally get

$$-\frac{1}{4}\mathrm{Tr}\left[\gamma_5\gamma^0 G^0\right] \sim -8\; f_{A\pi A}\; f^2_{qAp}\; m\epsilon\;. \tag{A9}$$

Keeping in mind the $V - A \Leftrightarrow \gamma^\mu + \gamma_5\gamma^\mu$ structure of the current, we deduce that $f_{A\pi A}$ is a negative quantity and is parametrized as

$$f_{A\pi A}(P_A^2) = -N_{\pi A}\,\frac{P_A^2 - M_A^2}{|P_A^2 - \Lambda_\pi^2|^\alpha}\;. \tag{A10}$$

The last open problem is to determine the strength of this coupling, in other words the size of $N_{\pi A}$. We can assume that the pion emission, as it would occur in a pion exchange between, e.g., two protons, is of the same order for all constituents of the protons, irrespective of modelling the proton as an ensemble of three valence quarks or of a quark and a diquark. Therefore, we can compare the amplitudes corresponding to the quark-pion-quark vertex, $\mathcal{M}_{q\pi q}$, and to the diquark-pion-diquark vertex, $\mathcal{M}_{A\pi A}$, assuming that they represent part of the diagrams describing the pion exchange between the constituents (quark, diquark) of a proton and another particle. Using again the rules defined in Sec. III we can write

$$\begin{aligned}
|\mathcal{M}_{q\pi q}|^2 &= \mathrm{Tr}\left[f_{q\pi q}\gamma_5\frac{i}{\not k' - m} f_{q\pi q}\gamma_5\frac{i}{\not k - m}\right] \\
&= 4\frac{f^2_{q\pi q}}{(k'^2 - m^2)(k^2 - m^2)}(k\cdot k' - m^2) \\
|\mathcal{M}_{A\pi A}|^2 &= \mathrm{Tr}\left[-if_{A\pi A}\;\epsilon_{\mu\nu\alpha\beta}\;k^\alpha k'^\beta\;\frac{i}{k'^2 - M_A^2}(-g^{\nu\rho})(-if_{A\pi A})\;\epsilon_{\rho\sigma\alpha'\beta'}\;k'^{\alpha'}k^{\beta'}\;\frac{i}{k^2 - M_A^2}(-g^{\sigma\mu})\right] \\
&= 2\frac{f^2_{A\pi A}}{(k'^2 - M_A^2)(k^2 - M_A^2)}\left[(k\cdot k')^2 - M_A^4\right]\;,
\end{aligned} \tag{A11}$$

where $k^{(\prime)}$ is the incoming (outgoing) momentum in the vertex for both quark and diquark and the couplings $f_{q\pi q}, f_{A\pi A}$ are assumed pointlike for simplicity. If the virtuality of the pion is small, in the c.m. of, e.g., the initial quark with $k = (m, \vec{0})$ we can assume $k\cdot k' \simeq m\epsilon$, with $\epsilon \sim m$ some energy such that $\epsilon - m \ll m$. Similar arguments hold also for the diquark and are independent of the angular distribution of the pion. Inserting these relations in Eq. (A11), we get

$$|\mathcal{M}_{A\pi A}|^2 \sim |\mathcal{M}_{q\pi q}|^2 \implies f_{A\pi A} \sim \frac{f_{q\pi q}}{M_A}\;. \tag{A12}$$

If the couplings are not pointlike and have the structure shown in Sec. III B, we need to search for a scale k_0^2 at which they are comparable functions of k^2, i.e.

$$|f_{A\pi A}(k_0^2)| = N_{\pi A}\frac{k_0^2 - M_A^2}{|k_0^2 - \Lambda_\pi^2|^2} \sim \frac{|f_{q\pi q}(k_0^2)|}{M_A} = \frac{N_{q\pi}}{M_A}\frac{k_0^2 - m^2}{|k_0^2 - \Lambda_\pi^2|^{3/2}}\;, \tag{A13}$$

with $\Lambda_\pi = 0.4$ GeV and $N_{q\pi} = 2.564$ GeV [9]. This is true for $N_{A\pi A} = 6$ GeV and $1 < k_0^2 < 2$ GeV^2, which is reasonably far from the pole Λ_π and from the asymptotic scale where the two form factors have very different power laws.

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